February 3, 1993 WOCOMAL VARSITY MEET

ROUND I: Combinations and permutations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AS POSITIVE INTEGERS

1. A computer operator must select and choose an order for four jobs from among ten waiting to be finished. How many different arrangements are possible?

2 If $\frac{{}_nP_5}{{}_nP_3} = 42$, find n

3 A space shuttle crew has available 10 main dishes, 8 vegatable dishes, 13 desserts, and 3 appetizers. If the first meal includes 2 desserts and one item from each of the other categories, how many different combinations are possible?

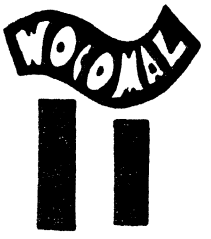
ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Shrewsbury, South



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ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. I have sold $\frac{2}{3}$ of my pencils for 15¢ each. If I have 8 pencils left, how much money did I collect for the pencils I sold?

2. Nine copies of a certain pamphlet cost less than \$10.00, while ten copies of the same pamphlet at the same price cost more than \$11.00. How much does one copy of this pamphlet cost?

3. Solve:

$$1 = \frac{1 + \dfrac{1 + \dfrac{1 + \frac{x}{4}}{3}}{2}}{1 + \dfrac{2}{1 + \dfrac{3}{1 + \frac{4}{5}}}}$$

ANSWERS

(1 pt) 1. \$ _____

(2 pts) 2. \$ _____

(3 pts) 3. $x =$ _____

Bromfield, Burncoat, Worcester Academy

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ROUND III: Logarithms, exponents, radicals

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1 Simplify : $\dfrac{2^{n+4} - 2(2^n)}{2(2^{n+3})}$.

2. If $a * b = a^b$ and $a \nabla b = \sqrt[b]{a}$,

evaluate $\left[(2*6) \nabla 3\right] * \left(-\frac{1}{2}\right)$.

3. Solve for x :

$$x^2 \log_{10} 8 - x \log_{10} 5 = 2 (\log_2 10)^{-1} - x$$

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. $x =$ _____

Bromfield, South, Tahanto

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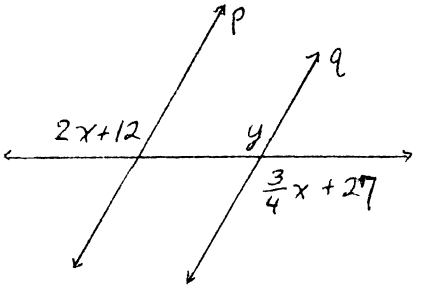
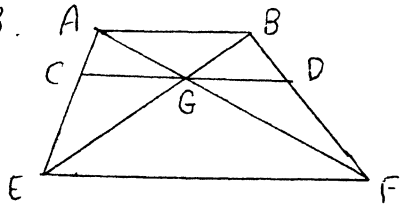
ROUND IV: Parallel lines and polygons

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. 

If $p \parallel q$ and the expressions represent angle measures, find y.

2. B and D are midpoints of $\overline{AC}$ and $\overline{EC}$ in $\triangle ACE$. $BD = 7$. The perimeter of $\triangle BDC$ is 20. Find the perimeter of ABDE.

3. 

If $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$, $AB = 10$, and $EF = 20$, find CD.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Millbury, Worcester Academy, QSC

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ROUND V: Analytic geometry of straight lines and conic sections

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. An isosceles trapezoid ABCD has $A(-4a, 0)$ and $B(4a, 0)$ where $a > 0$. The height of the trapezoid is h and the upper base $\overline{DC}$ has length $2c$. Find the coordinates of vertex D (which is in the second quadrant) in terms of letters already stated.

2. Find the coordinates of the point(s) on the line $y = 3x + 1$ that is (are) equidistant from $(0,0)$ and $(-3, 4)$.

3. Find all points (x, y) for which the x-coordinate is twice the y-coordinate and which lie on the circle of radius 5 with center at $(2, 6)$.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

St. John's. Shrewsbury, Tantasqua

February 3, 1993 WOCOMAL VARSITY MEET

TEAM ROUND: Topics of previous rounds and open

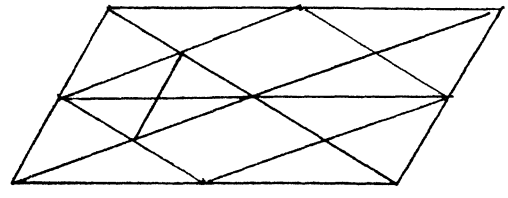
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ROUND ANSWER SHEET 2 points each

1. The 7 digits zero through six are arranged on a line at random with no repeats. In how many ways could you form an even number greater than three million? Give your answer as an integer.

2. If $P = \frac{A}{B}$, $Q = \frac{B}{C}$, and $R = \frac{C}{A}$, express $\frac{A+B}{A+C}$ as a single simplified fraction in terms of P and R.

3. Solve $\log_{10}(x^2-1) = \log_{10}(x-1) + \frac{1}{2}$ for x.

4. Assuming parallel lines and concurrence from the diagram, how many parallelograms are shown here?



5. Find the exact distance between the two foci of the ellipse $x^2 + 4y^2 = 32$.

6. If $A = \begin{bmatrix} .\overline{12} & .\overline{45} \\ .\overline{04} & .\overline{72} \end{bmatrix}$ and $B = \begin{bmatrix} 99 \\ 11 \end{bmatrix}$, determine the product AB.

7. If $(x+y)^2 = 40$ and $(x-y)^2 = 25$, evaluate $2x^2 - 40xy + 2y^2$.

8. $f(x)$ is a monomial in x and $f(f(x)) = 4x^4$. Find $f(x)$.

9. How many fractions $\frac{a}{b}$ are there such that a and b are integers, $0 < a < b$, $\frac{a}{b}$ is in lowest terms, and b divides 24? Note: b could $= 24$.

Algonquin, Doherty, Hudson, Leicester, Quaboag, Tahanto, Worcester Academy, QSC

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ROUND I comb. perm.	1pt	1.	5,040
	2pts	2.	10
	3pts	3.	18,720
ROUND II alg 1	1pt	1.	*2.40
	2pts	2.	*1.11
	3pts	3.	$x=2$
ROUND III logs exp radicals	1pt	1.	$\frac{7}{8}$
	2pts	2.	$\frac{1}{2}$
	3pts	3.	$x=-1$ or $\frac{2}{3}$ need both
ROUND IV 11 inner polygons	1pt	1.	36
	2pts	2.	34
	3pts	3.	$\frac{40}{3}$ or $13\frac{1}{3}$ or $13.\overline{3}$
ROUND V analytic geom	1pt	1.	$(-c, h)$
	2pts	2.	$\left(\frac{17}{18}, \frac{23}{6}\right)$ 2nd coord could be $3\frac{5}{6}$
	3pts	3.	$(2,1)$, $(6,3)$ need both

TEAM ROUND
2pts each

1. 1680
2. $\frac{P+1}{P(R+1)}$ OR $\frac{P+1}{PR-P}$
3. $\sqrt{10}-1$
4. 18
5. $4\sqrt{6}$
6. $\begin{bmatrix} 17 \\ 12 \end{bmatrix}$
7. -85
8. $3\sqrt{4}\,x^2$ or $2^{\frac{2}{3}}x^2$
9. 23